MULTIFUNCTIONAL COLLABORATIVE MODELING AND ANALYSIS METHODS IN ENGINEERING SCIENCE

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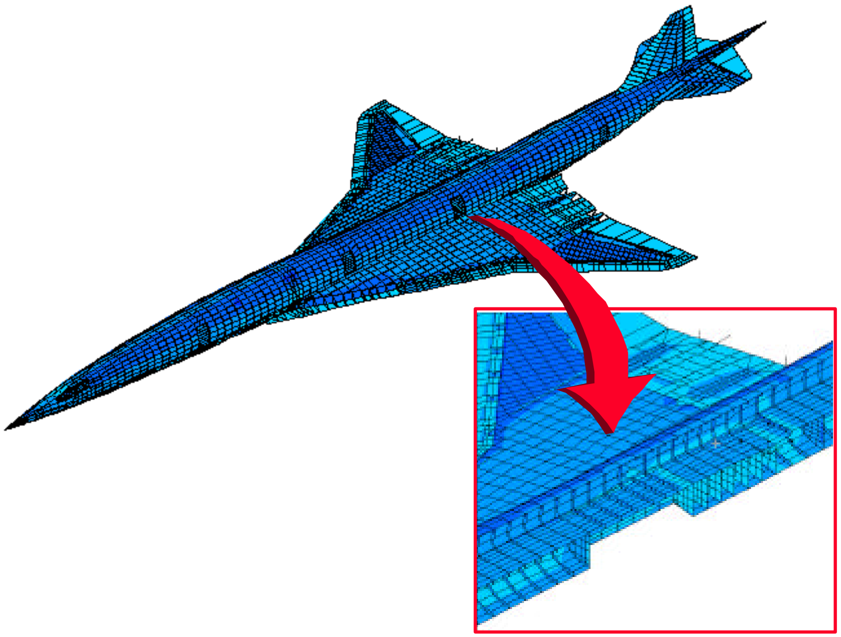
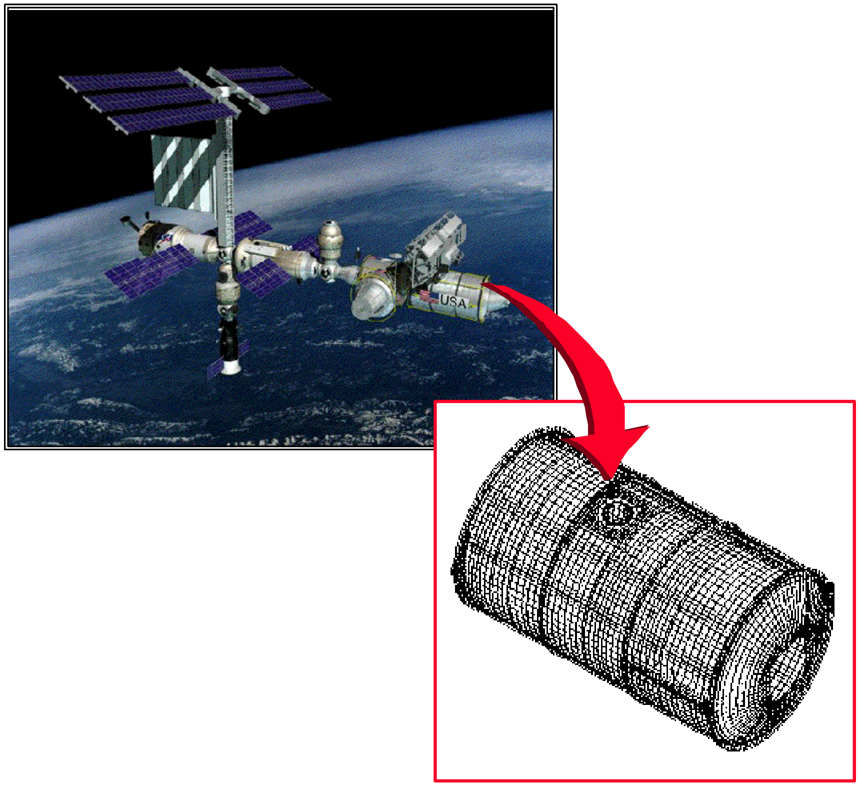
CONVENTIONAL MODELING AND ANALYSIS OF COMPLEX SYSTEMS





- Multi-fidelity FE Modeling
- Requires months to model
- Changes expensive, time consuming and error prone
- Model often tied to discipline

Multifunctional Collaborative Methodology

- Multi-fidelity
- Multiple Methods
- Multiple Disciplines

OBJECTIVES

- **Present general methodology providing capability for multifunctional modeling, analysis and solution**
- **Identify computational aspects and related algorithmic issues for this methodology**
- **Demonstrate the formulation to scalar- and vector-field applications in engineering science**

KEY TERMINOLOGY

- **Multifunctional** - Computational methodologies for rapid, robust solutions featuring multi-fidelity modeling and multiple methods
- **Collaborative** - Mechanism by which two or more physical domains or methods are integrated/interfaced
- **Engineering science** - Broad spectrum of engineering (science, mathematics, numerical analysis)
- **Homogeneous modeling** - Same spatial modeling approach among subdomains
- **Heterogeneous modeling** - Different spatial modeling approaches among subdomains

Example - Previous interface technology demonstrated collaborative method for homogeneous FE modeling applied to solid mechanics

MULTIFUNCTIONAL METHODOLOGY

Multifunctional Methods Concept and General Formulation

Homogeneous Modeling | **Heterogeneous Modeling**

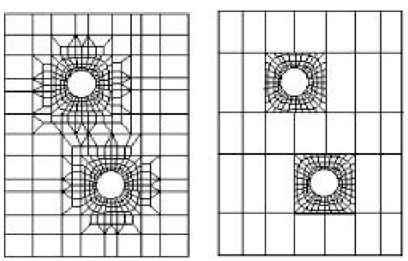
FEM Model Interfacing

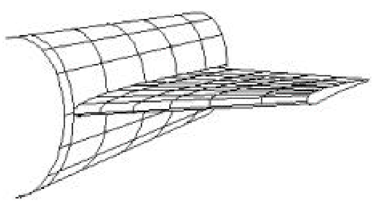
Non-FEM Model Interfacing

Multiple Method Interfacing

Multiple Discipline Interfacing

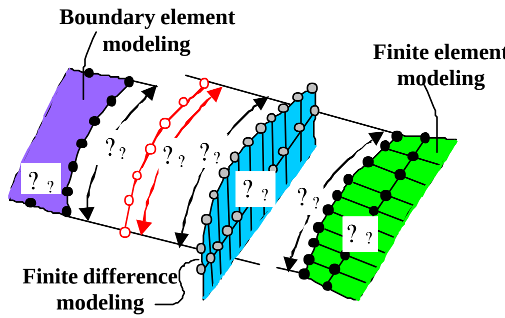
Multi-fidelity

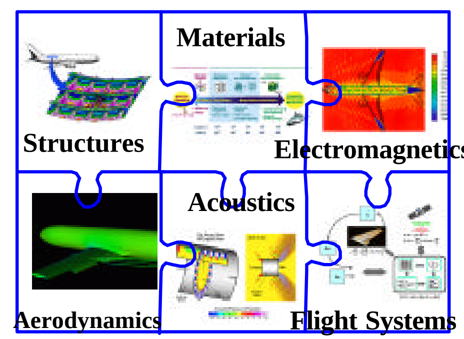




Multiple-domain

Multiple methods





Multiple disciplines

Capability which Enables the Synthesis of Diverse Models

OUTLINE

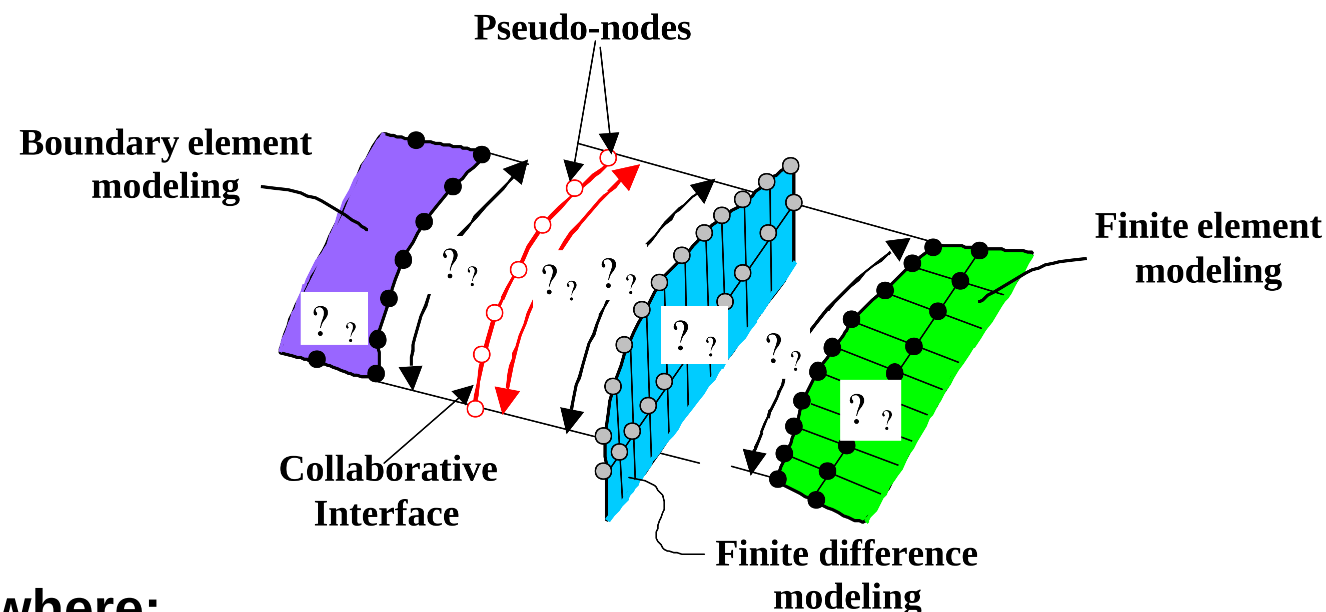
- **Multifunctional Formulation**
  - Basic assumption
  - Method of weighted residuals for MFC approach
  - Collaborative interface treatment
  - General system of equations
- **Selected Applications**
- **Concluding Remarks**

MULTIFUNCTIONAL FORMULATION

-BASIC ASSUMPTION-

Deformation, v, along the interface connecting the substructures, ? $_i$, may be expressed as:

$$\mathbf{v} = \mathbf{T}\mathbf{q}_{\mathrm{I}}$$



where:

- **T is an interpolation matrix formed using cubic splines**
- **$\mathbf{q}_{\mathrm{I}}$ is a vector of interface degrees of freedom**

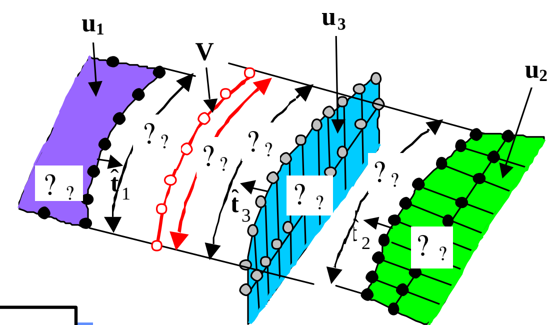
MULTIFUNCTIONAL FORMULATION

- METHOD OF WEIGHTED RESIDUALS -

Define:

$$\overline{\overline{\mathbf{R}}} ? \overset{N_{ss}}{\underset{m?1}{?}} \overline{\mathbf{R}}_m ? \overset{N_{\mathrm{I}}}{\underset{i?1}{?}} ?\overline{\mathbf{R}}_{c_i^s} ? \overline{\mathbf{R}}_{c_i^p} ?? 0$$

where the orthogonalized residuals associated with:



Governing equation within the domain

$$\overline{\mathbf{R}}_m ? \underset{?_m}{?} \mathbf{F}_m \mathbf{R}_m \, \mathrm{d}?_m ? \mathbf{0}$$

Compatibility constraint for <u>primary variable</u> on interface boundary

$$\overline{\mathbf{R}}_{c_i^p} ? \overset{n_i}{\underset{j?1}{?}} \underset{?_{\mathrm{I}}}{?} \mathbf{?}_j ?\mathbf{v}_i ? \mathbf{u}_j ? \mathrm{d}?_{\mathrm{I}} ? \mathbf{0}$$

Compatibility constraint for <u>secondary variable</u> on interface boundary

$$\overline{\mathbf{R}}_{c_i^s} ? \overset{n_i}{\underset{j?1}{?}} \underset{?_{\mathrm{I}}}{?} \hat{\mathbf{?}}_i \hat{\mathbf{t}}_j \, \mathrm{d}?_{\mathrm{I}} ? \mathbf{0}$$

MULTIFUNCTIONAL FORMULATION
- COLLABORATIVE INTERFACE TREATMENT -

For weight functions:

FE domains: $\mathbf{F}_m \; ? \; \mathbf{N}_m$

FD domains : $\mathbf{F}_m \; ? \; ? \; ?x ? x_l , y ? y_l ? ? \; ? \; ?x_l , y_l ?$

Assume for each interface *i*:

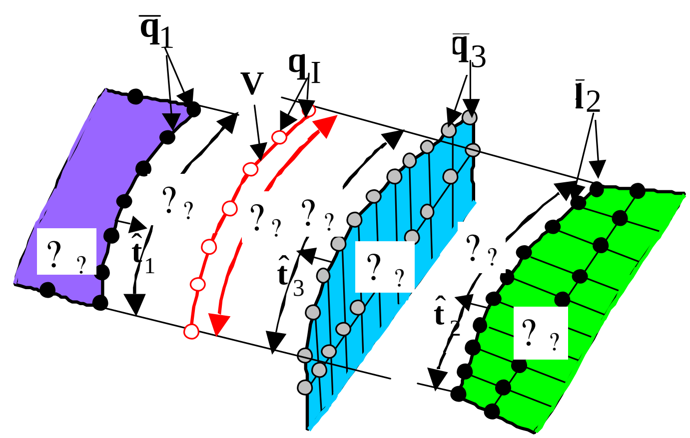
$\mathbf{u}_j \; ? \; \mathbf{N}_j \bar{\mathbf{q}}_j , \quad \mathbf{v}_i \; ? \; \mathbf{T}\mathbf{q}_{\mathrm{I}} , \quad \hat{\mathbf{t}}_j \; ? \; \mathbf{S}_j \mathbf{a}_j \quad \hat{\mathbf{?}}_i \; ? \; \mathbf{T},$ and $\mathbf{?}_j \; ? \; \mathbf{S}_j$

substituted into:

$$\bar{\mathbf{R}}_m \; ? \; \underset{?_m}{?} \mathbf{F}_m \mathbf{R}_m \, \mathrm{d}?_m \; ? \; 0$$

$$\bar{\mathbf{R}}_{c_i^{\mathrm{p}}} \; ? \; \underset{j?1}{\overset{n_i}{?}} \; \underset{?_{\mathrm{I}}}{?} ?\mathbf{?}_j ?\mathbf{v}_i \; ? \; \mathbf{u}_j ? \mathrm{d}?_{\mathrm{I}} \; ? \; 0$$

$$\bar{\mathbf{R}}_{c_i^{\mathrm{s}}} \; ? \; \underset{j?1}{\overset{n_i}{?}} \; \underset{?_{\mathrm{I}}}{?} ?\hat{\mathbf{?}}_i \hat{\mathbf{t}}_j \, \mathrm{d}?_{\mathrm{I}} \; ? \; 0$$



MULTIFUNCTIONAL FORMULATION

- GENERAL SYSTEM OF EQUATIONS -

General matrix form for multifunctional collaborative approach

For scalar-fields

$$\begin{bmatrix} \mathbf{K} & \mathbf{0} & \mathbf{K}_s \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_I \\ \mathbf{K}_p & \mathbf{K}_I^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \mathbf{q}_I \\ \mathbf{a} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}$$

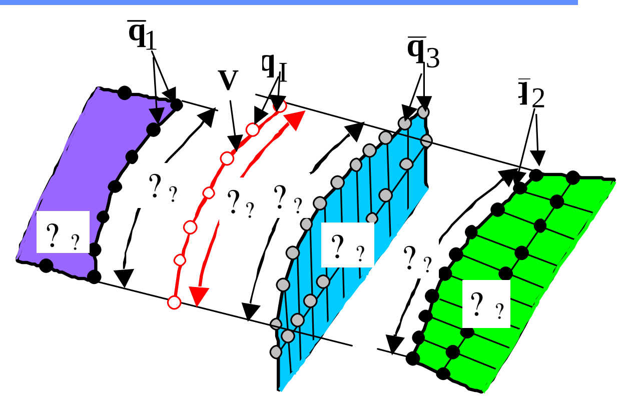
For vector-fields

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{q}}_I \\ \bar{\mathbf{a}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{0} & \bar{\mathbf{K}}_s \\ \mathbf{0} & \mathbf{0} & \bar{\mathbf{K}}_I \\ \bar{\mathbf{K}}_p & \bar{\mathbf{K}}_I^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \mathbf{q}_I \\ \bar{\mathbf{a}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}$$

where $\bar{\mathbf{K}}_s, \bar{\mathbf{K}}_p, \bar{\mathbf{K}}_I^T, \bar{\mathbf{a}}$ are dependent on fluid mechanics formulation and $\mathbf{u}$ is discipline-specific

Matrix characteristics:

- Sparse
- Non-positive definite
- May be unsymmetric

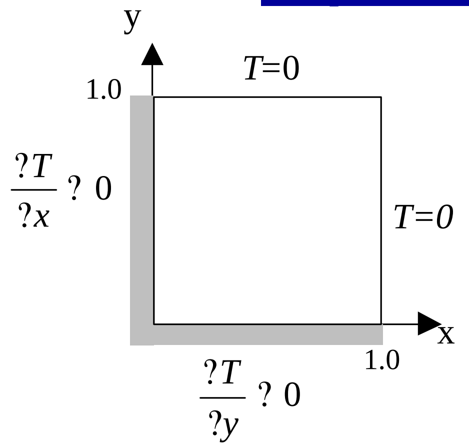


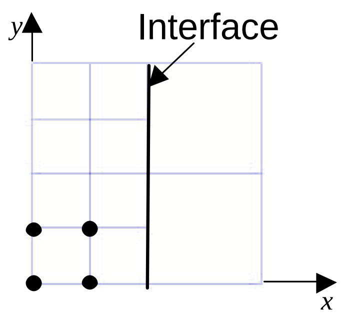
APPLICATIONS

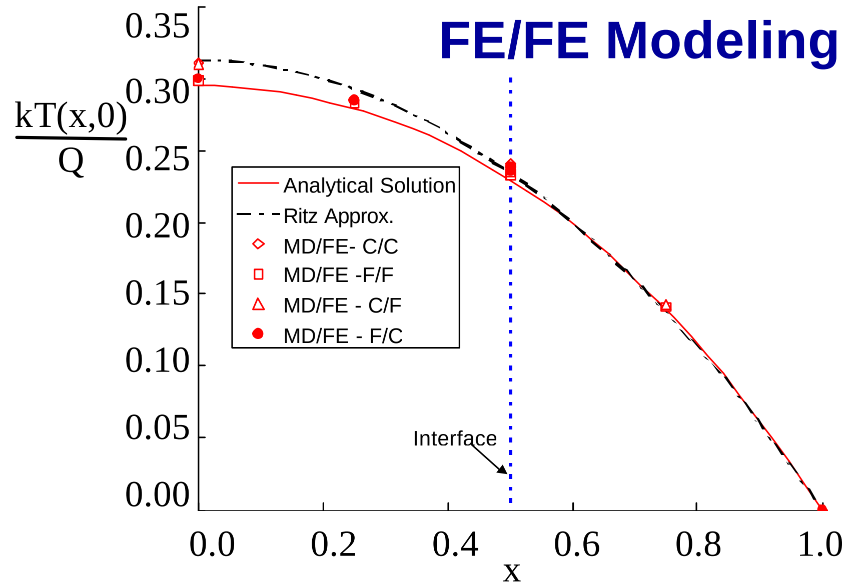
- Patch Test Example Problems
- Torsion of Prismatic Bar
- Heat Conduction Problem
- Potential Flow Problem
- Plane Stress Problem
- Plane Flow Problem
- Boeing Crown Panel
- Douglas Stub-Box

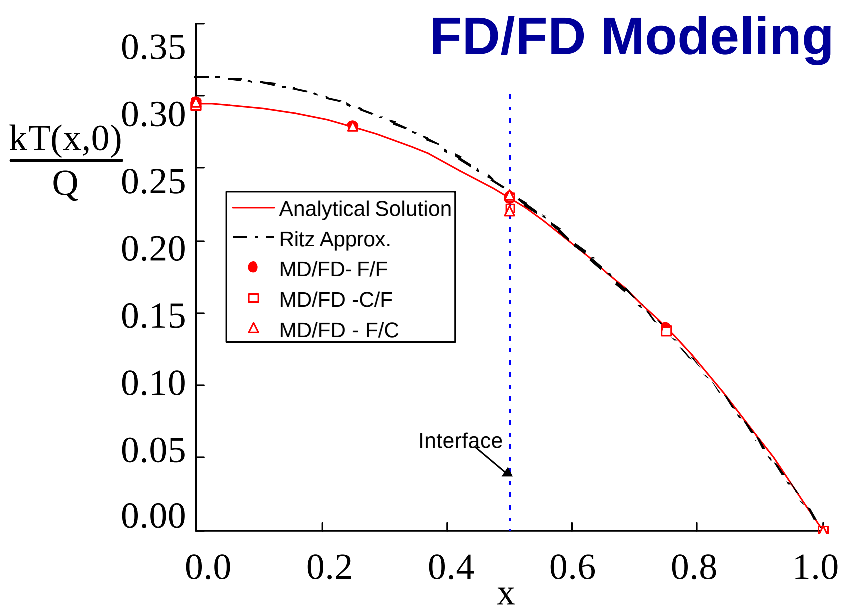
HEAT CONDUCTION PROBLEM

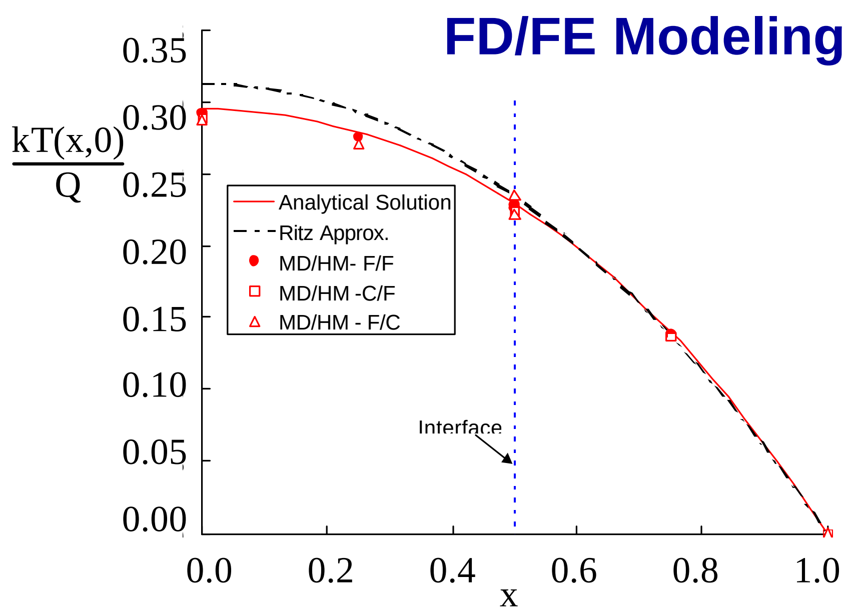
Square Plate





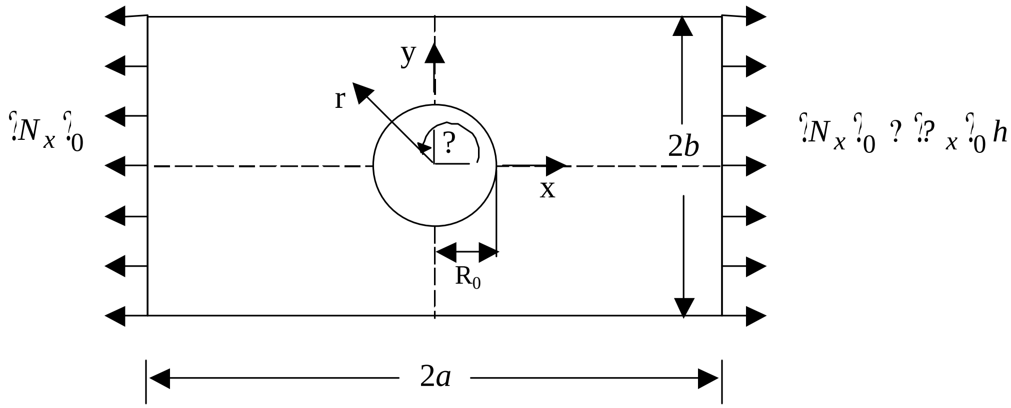






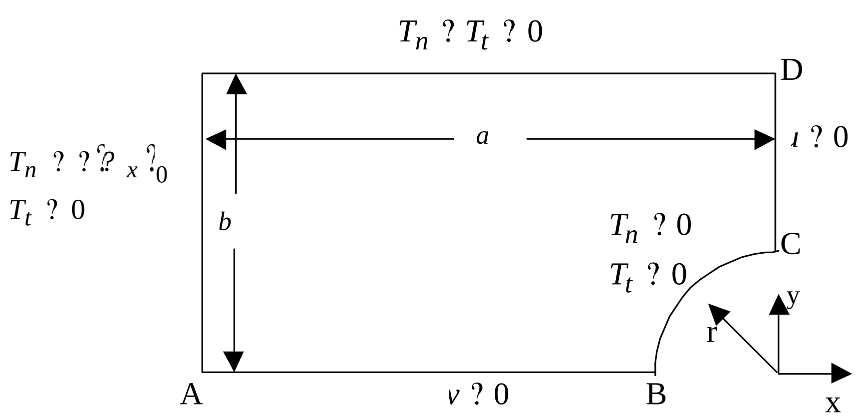
PLANE STRESS PROBLEM

Plate with Central Cutout



E=10,000 ksi
?=0.3
h=0.1 in.

Geometric Configuration



Two Configurations:

Infinite plate:

$2a/R_0$ = 40

$2b/R_0$ = 20

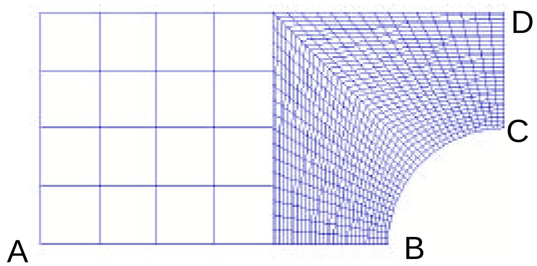
Finite-width plate:

$2a/R_0$ = 4

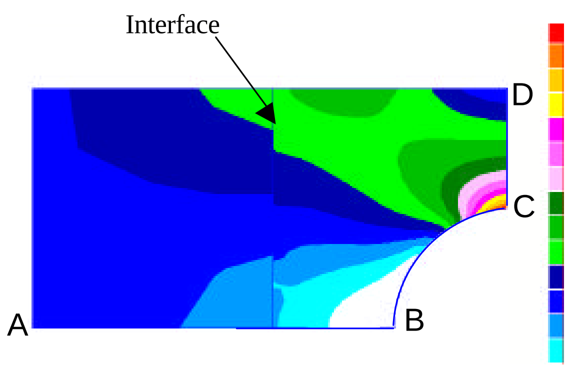
$2b/R_0$ = 2

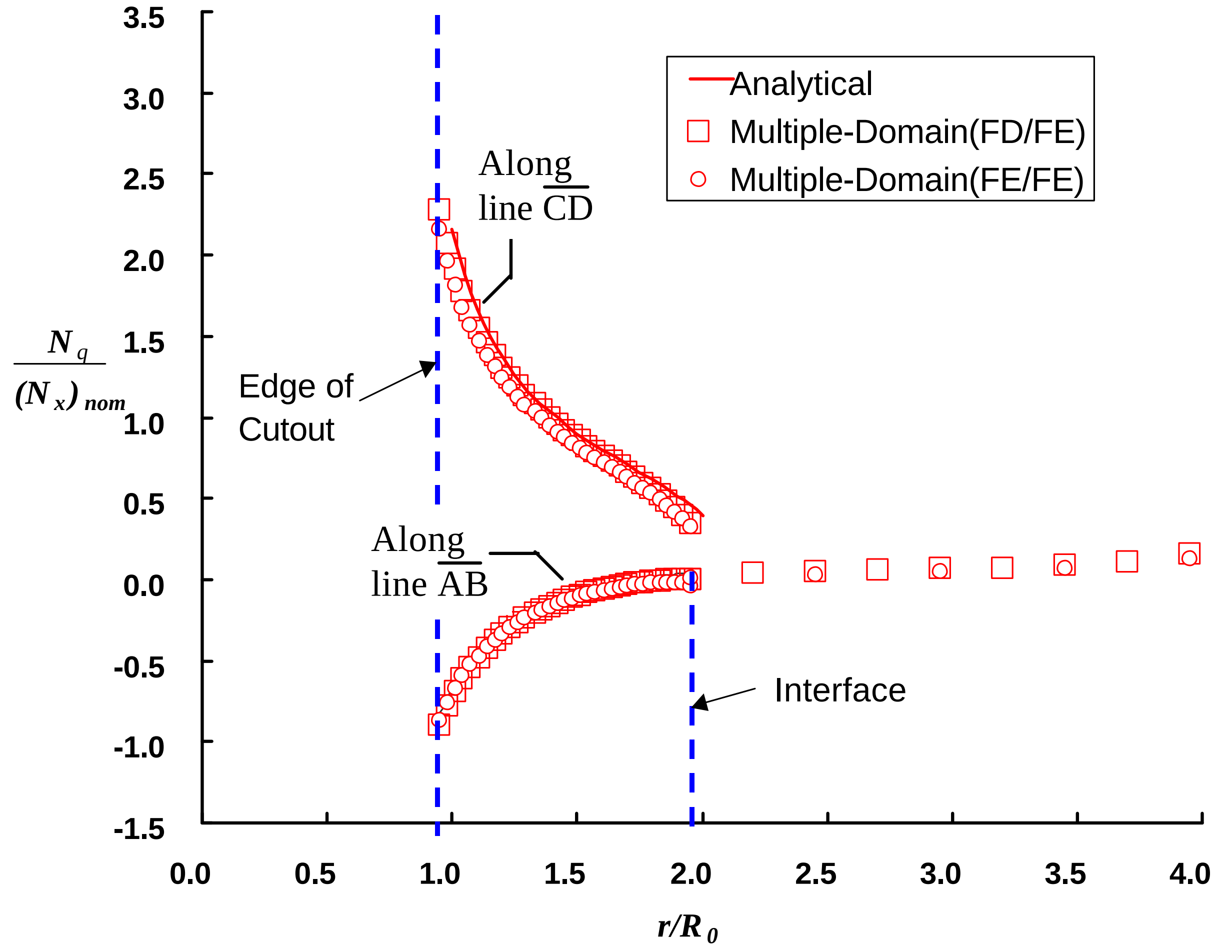
STRESS RESULTANT DISTRIBUTION FOR FINITE-WIDTH PLATE

Spatial Modeling



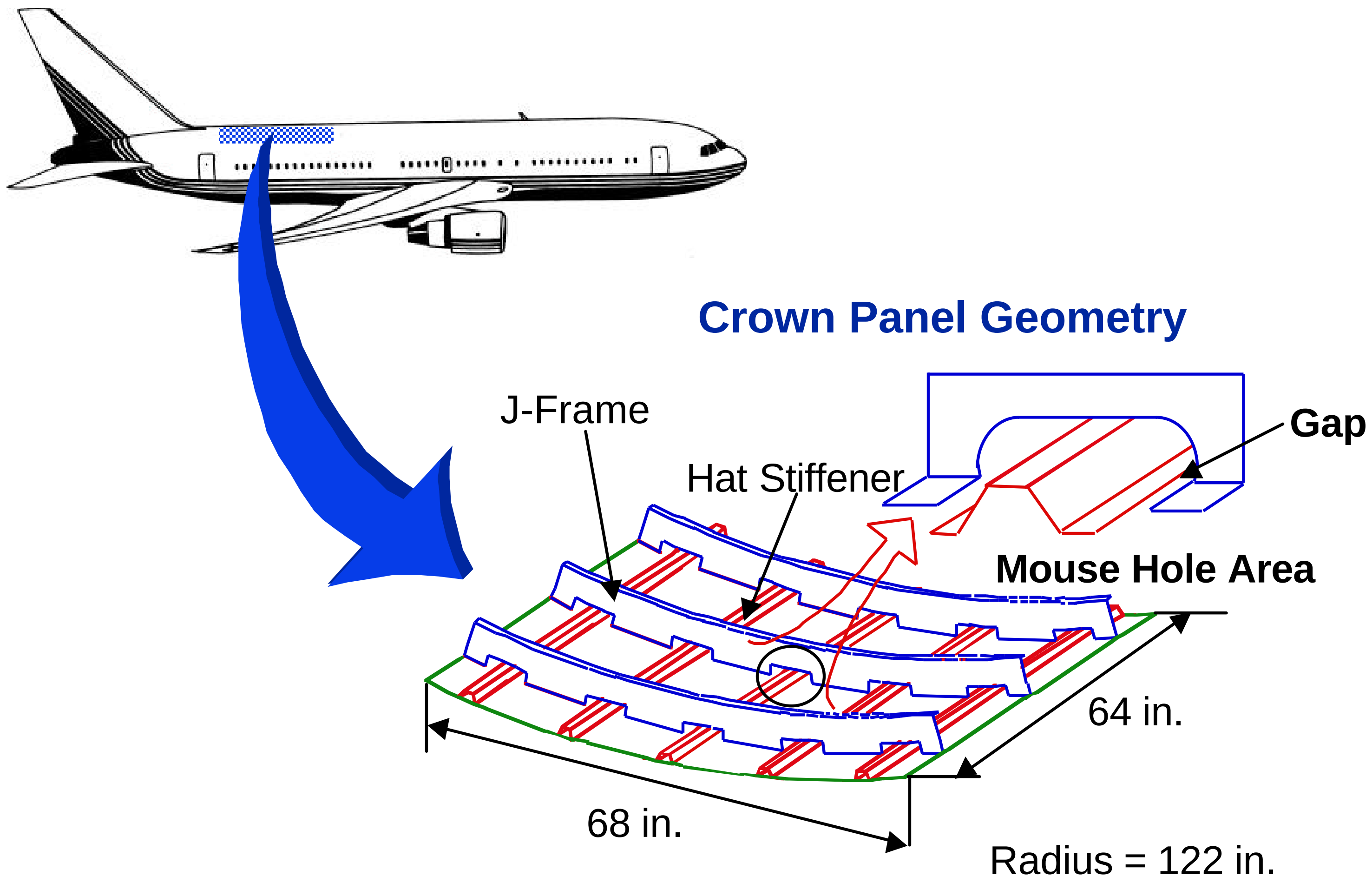
Stress Resultant Contours





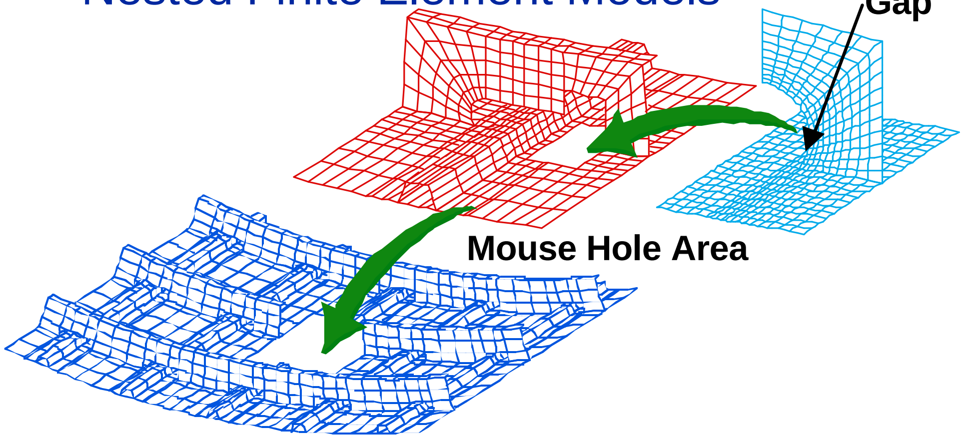
$$A_{net} = (2b - 2R_0)h = 2bh\left(1 - \frac{R_0}{b}\right) \quad ; \quad (\sigma_x)_{nom} = \frac{P}{A_{net}} \quad ; \quad (N_x)_{nom} = (\sigma_x)_{nom}\, h$$

COLLABORATIVE METHODOLOGY DEMONSTRATED ON BOEING CROWN PANEL

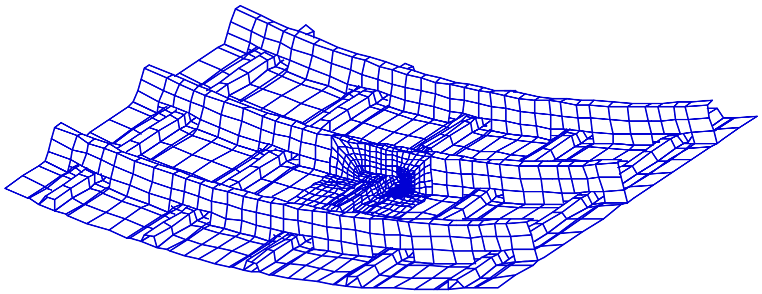


COLLABORATIVE METHODOLOGY DEMONSTRATED ON BOEING CROWN PANEL

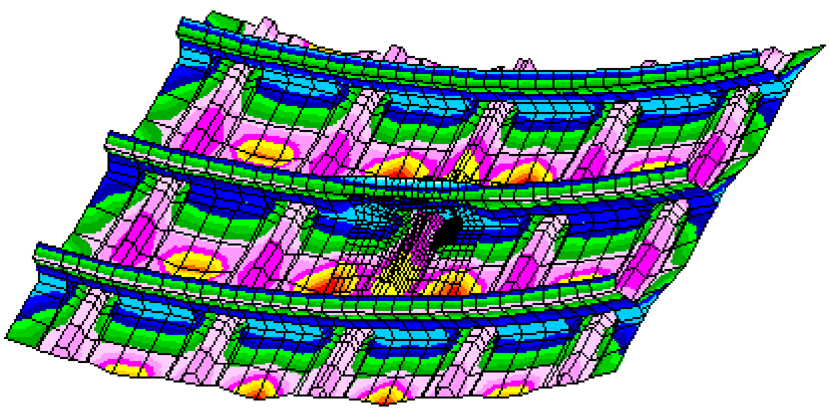
Nested Finite Element Models



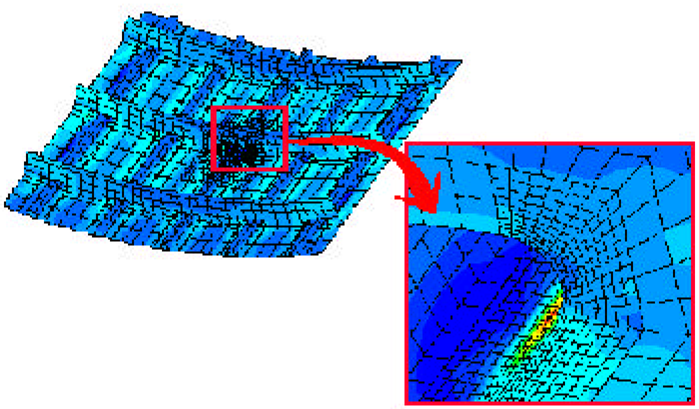
Combined Finite Element Model



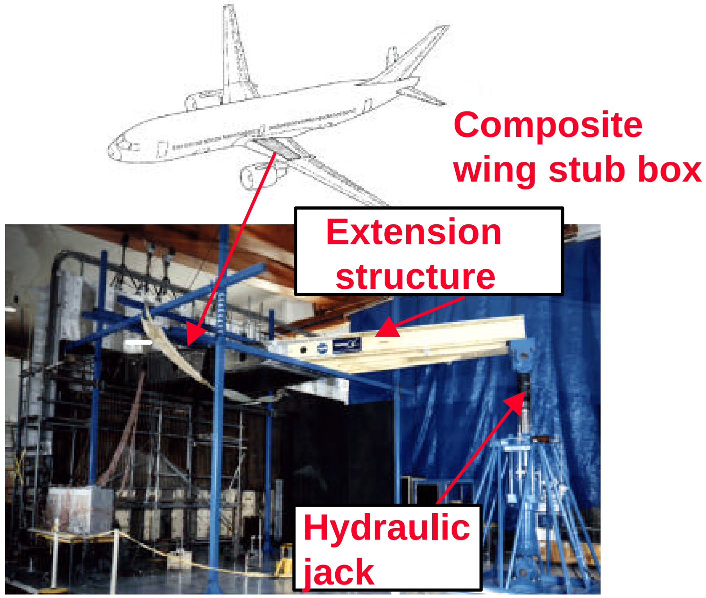
Deformation Contour



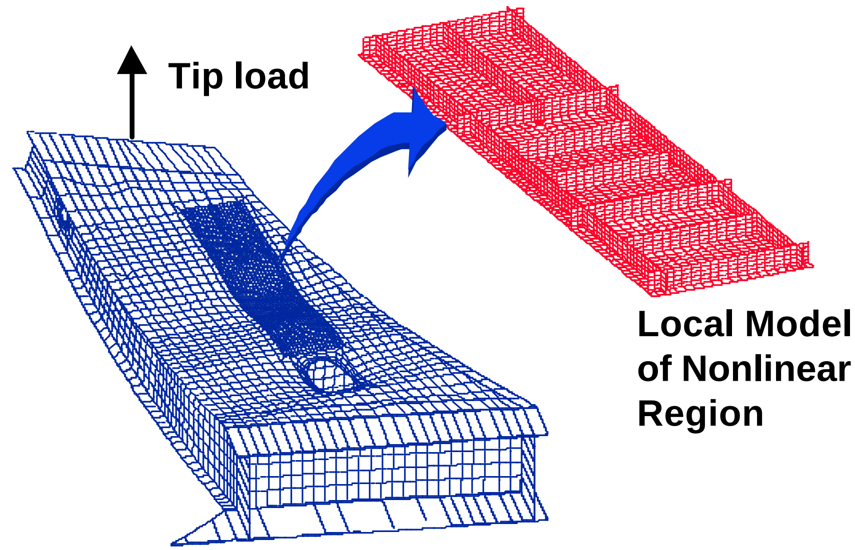
Hoop Stress Contour



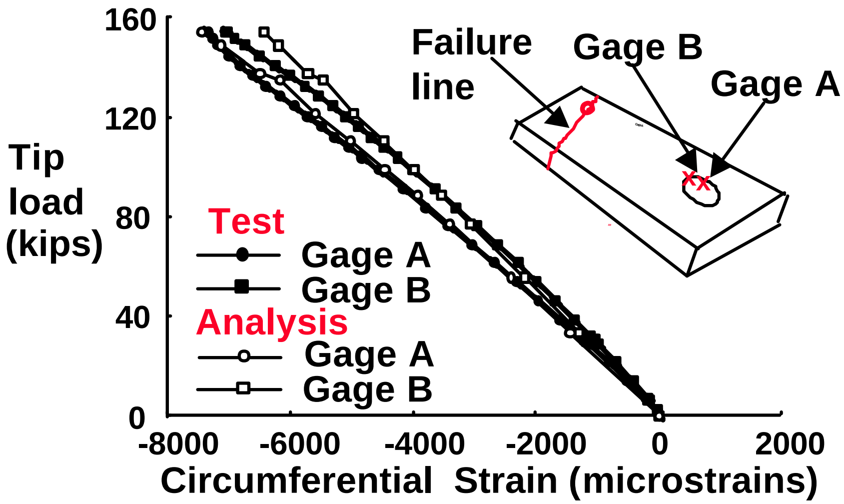
APPLICATION OF COLLABORATIVE METHODOLOGY IN NONLINEAR ANALYSIS OF WING STUB BOX



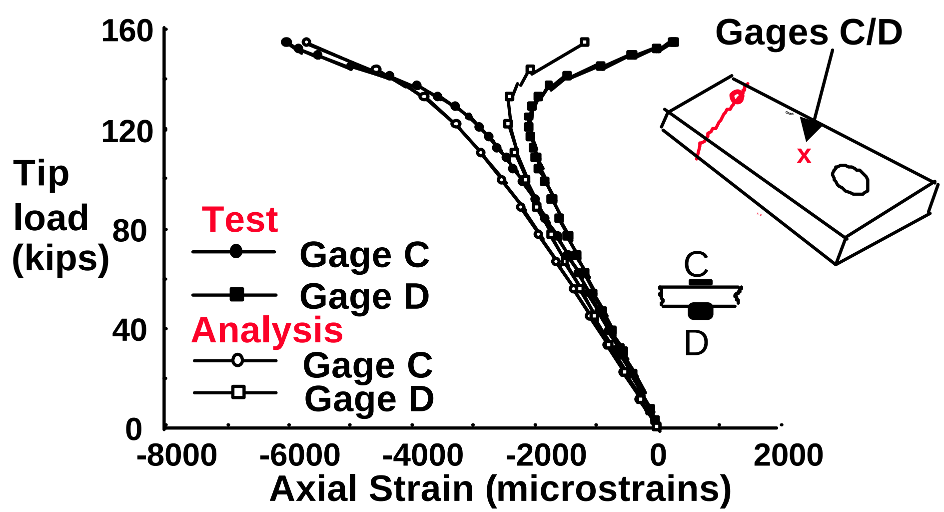
Experimental Setup



Deformed Shape



Access Door Cutout



Nonlinearly Deformed Unstiffened Bay

SUMMARY

- Results presented for patch test, scalar-field, and vector-field problems
- Results for all problems and multifunctional approaches in overall good agreement
- Finite element solutions more accurate than finite difference solutions for discretizations considered
- Results with heterogeneous modeling not as accurate as homogeneous modeling

CONCLUSIONS

- Multifunctional collaborative methodologies and analysis procedures formulated and placed on solid mathematical foundation
  - Scalar-field and vector-field problems
  - Homogeneous and heterogeneous modeling
- Collaborative role of modeling approaches has been illustrated
- Capabilities demonstrated on benchmark problems and large scale applications
- Computational challenges overcome
- Application of FD method limits general use